Solution to MHT CET – 2021

24^{th} September (Shift - 2)

Section I

PHYSICS

1. (D)

$$n\phi = LI$$

$$\therefore \frac{LI}{n} = \phi$$

2. (A)

3. (D)

Intensity is proportional to the number of photons, since each photon interacts with one electron, the number of electrons $N \propto I$.

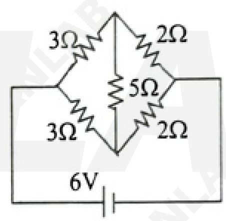
Energy of a photon is $\frac{hc}{\lambda}$, hence it is inversely proportional to the wavelength.

Hence the energy received by the electron

$$E \propto \frac{1}{\lambda}$$

4. (A)

The given circuit can be redrawn as shown in the figure.



It is a balanced Wheatstone bridge. No current will flow through 5 Ω resistance, and hence it can be removed from the circuit. 3 Ω and 2 Ω resistances are in series. Hence we have two branches with 5 Ω resistances each, connected in parallel. Their equivalent resistance is 2.5 Ω

$$\therefore I = \frac{V}{R} = \frac{6}{2.5} = 2.4 \text{ A}$$

5. (B)

The moment of inertia of the rod about an axis passing through the centre and perpendicular to its length is given by

$$I_0 = \frac{ML^2}{12}$$

A point at a distance $\frac{L}{4}$ from its end will also be at a distance $\frac{L}{4}$ from the centre.

Hence by parallel axis theorem,

$$I = I_0 + M\left(\frac{L}{4}\right)^2 = \frac{ML^2}{12} + \frac{ML^2}{16} = \frac{7ML^2}{48}$$

6. (A)

Induced emf $e = M\frac{dI}{dt} = M \times 1000\pi\cos(100\pi t)$

$$= e_0 \cos 100\pi t$$

The maximum value of emf $e_0 = 1000\ \pi M$

$$\therefore M = \frac{5}{1000} = 5 \text{ mH}$$

7. (B)

$$g = \frac{GM}{R^2} = \frac{G}{R^2} \cdot \frac{4}{3}\pi R^3 \cdot \rho = G\frac{4}{3} \cdot \pi R \rho$$

$\therefore$ $g \propto \rho$

8. (C)

The frequency of oscillation is given by

$$f_P = \frac{1}{2\pi}\sqrt{\frac{M_P B}{I_P}} \text{ and } f_Q = \frac{1}{2\pi}\sqrt{\frac{M_Q B}{I_Q}}$$

$$f_P = 2f_Q$$

$$\therefore \frac{1}{2\pi}\sqrt{\frac{M_P B}{I_P}} = 2 \cdot \frac{1}{2\pi}\sqrt{\frac{M_Q B}{I_Q}}$$

$$\therefore \frac{M_P}{I_P} = 4\frac{M_Q}{I_Q}$$

$$\therefore \frac{M_P}{M_Q} = 4\frac{I_P}{I_Q} = 8 \qquad [\because I_P = 2I_Q]$$

9. (A)

C_P and C_v are related as : $C_P = C_V + R$

Hence expression in (A) is not correct.

10. (D)

R.M.S speed is given by, $c = \sqrt{\frac{3RT}{M}}$

$T_{He} = 327°C = 600 \text{ K}, T_O = 27°C = 300 \text{ K}$

$$\therefore \frac{C_{He}}{C_O} = \sqrt{\frac{T_{He}}{T_o} \cdot \frac{M_O}{M_{He}}} = \sqrt{\frac{600}{300} \cdot \frac{32}{4}}$$

$C = 4$

11. (C)

In isochoric process, volume remains constant.

12. (A)

The potential energy of the magnetic dipole is given by

$U = -MB\cos\theta$

Where θ is the angle between $\vec{M}$ and $\vec{B}$

For $\theta = 0°$, $U = -MB\cos 0° = -MB$ (Minimum)

For $\theta = 180°$, $U = -MB\cos 180° = MB$ (Maximum)

13. (A)

At resonance, the net reactance of the circuit is zero and the impedance is equal to the resistance.

$$\therefore I_0 = \frac{E_0}{R}$$

14. (D)

Centripetal acceleration is given by

$$a = \frac{v^2}{r} \quad \therefore \; a \propto v^2$$

$$\therefore \quad \frac{a_2}{a_1} = \left(\frac{v_2}{v_1}\right)^2 = (2)^2 = 4$$

15. (C)

$x = 5 \sin(3t + 3)$

Standard equation of S.H.M. is $x = A \sin(\omega t + \alpha)$

$\therefore \; A = 5$ cm, $\omega = 3$ rad/s

Maximum acceleration $a_m = A\omega^2 = 5 \times (3)^2$
$= 5 \times 9 = 45 \text{ cm s}^{-2}$

16. (D)

The fundamental frequency of open tube is

$$n_1 = \frac{v}{2\ell_1}$$

When tube is dipped in water, one–fourth of it is in water and three–fourth is in air.

Hence, it becomes a tube closed at one end with length $\ell_2 = \frac{3}{4}\ell_1$

The fundamental frequency of closed tube is

$$n_2 = \frac{v}{4\ell_2}$$

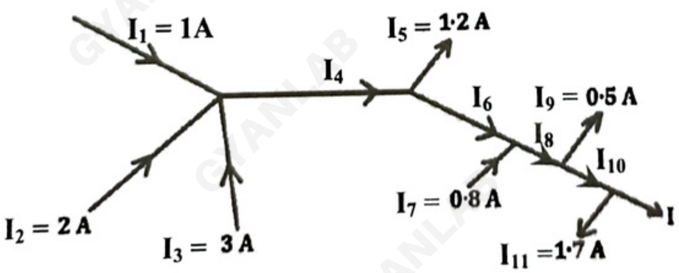
$$\therefore \quad \frac{n_2}{n_1} = \frac{1}{4\ell_2} \times 2\ell_1 = \frac{\ell_1}{2\ell_2}$$

$$= \frac{4}{2 \times 3} \quad \left[\because \frac{\ell_1}{\ell_2} = \frac{4}{3}\right]$$

$$= \frac{2}{3} n$$

$$\therefore \quad n_2 = \frac{2}{3} n_1 = \frac{2}{3} n$$

17. (C)



$I_4 = I_1 + I_2 + I_3 = 1 + 2 + 3 = 6$ A

$I_6 = I_4 - I_5 = 6 - 1.2 = 4.8$ A

$I_8 = I_6 + I_7 = 4.8 + 0.8 = 5.6$ A

$I_{10} = I_8 - I_9 = 5.6 - 0.5 = 5.1$ A

$I = I_{10} - I_{11} = 5.1 - 1.7 = 3.4$ A

18. (C)

When 40% nuclei decay, 60% nuclei remain undecayed

Let this number be N_0

When 85% nuclei decay, 15% nuclei remain undecayed

This number will be $N = \frac{N_0}{4}$

The number of nuclei will become one–fourth in two half lives i.e. 60 minutes.

19. (D)

$\frac{I_1}{I_2} = \frac{I}{4I} = \frac{1}{4} = \frac{a_1^2}{a_2^2}$

where a_1 and a_2 are the amplitudes

$\therefore \frac{a_1}{a_2} = \frac{1}{2}$ $\qquad \therefore \frac{a_{max}}{a_{min}} = \frac{a_1 + a_2}{a_1 - a_2} = \frac{1.2}{1-2} = \frac{3}{-1}$

$\therefore \frac{I_{max}}{I_{min}} = \frac{a_{max}^2}{a_{min}^2} = \frac{9}{1}$

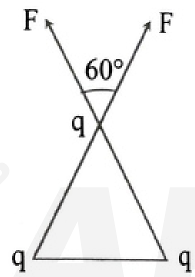
20. (D)

The force on any charge due to each of the other two charges has magnitude F.

The angle between the two forces is 60°.

Hence the resultant force is given by

$$R = \sqrt{F^2 + F^2 + 2F^2 \cos 60°} = \sqrt{3}F$$



21. (C)

Phase difference $\phi = \frac{2\pi x}{\lambda}$

Given $\phi = n\pi$; $\qquad \lambda = \frac{v}{f}$

$\therefore \qquad n\pi = \frac{2\pi x f}{v}$

$\therefore \qquad f = \frac{nV}{2x}$

22. (A)

23. (B)

$\beta_{dc} = \frac{\alpha_{dc}}{1 - \alpha_{dc}} = 69$

24. (B)

Volume $= \frac{4}{3}\pi r^3 n = \frac{4}{3}\pi R^3$

$\therefore \; R = n^{1/3} r \qquad \therefore \; \frac{R}{r} = n^{\frac{1}{3}}$

Terminal velocity $v \propto r^2$

$$\frac{v_2}{v_1} = \left(\frac{r_2}{r_1}\right)^2 = \left(\frac{R}{r}\right)^2 = n^{2/3}$$

$\therefore\ v_2 = n^{2/3} v_1 = 5\, n^{2/3}$ m/s

25. (B)

$$g' = g\left(1 - \frac{d}{R}\right)$$

$$g' = 0.6\,g \qquad \therefore\ 0.6 = 1 - \frac{d}{R}$$

$$\therefore\ \frac{d}{R} = 1 - 0.6 = 0.4 = \frac{2}{5}$$

$$\therefore\ d = \frac{2}{5}R$$

26. (C)

Centripetal force $F = \dfrac{mv^2}{r}$

$$F_1 = \frac{mv_1^2}{r_1} \text{ and } F_2 = \frac{mv_2^2}{r_2}$$

$$\therefore\ \frac{F_2}{F_1} = \frac{v_2^2}{v_1^2}\cdot\frac{r_1}{r_2}$$

$$v_2 = \frac{v_1}{2} \text{ and } r_2 = 3r_1$$

$$\therefore\ \frac{F_2}{F_1} = \left(\frac{1}{2}\right)^2\cdot\frac{1}{3} = \frac{1}{12}$$

$$\therefore\ F_2 = \frac{F_1}{12} \qquad \therefore\ F_2 < F_1$$

$$\therefore\ F_1 - F_2 = F_1 - \frac{F_1}{12} = \frac{11}{12}F_1$$

27. (D)

Path difference $= \dfrac{57}{2}\lambda = 28.5\lambda$

Since the path difference is not an integral multiple of λ, it is not a bright bond.

Path difference for nth dark band is given by $\left(n - \dfrac{1}{2}\right)\lambda$

Hence n = 29.

28. (B)

Excess pressure inside a soap bubble is given by

$$\Delta P = P_i - P_0 = \frac{4T}{r}$$

$\Delta P_1 = 1.01$ atm $- 1$ atm $= 0.01$ atm

$\Delta P_2 = 1.03$ atm $- 1$ atm $= 0.03$ atm

$\therefore \quad \frac{\Delta P_1}{\Delta P_2} = \frac{r_2}{r_1} \qquad \therefore \quad \frac{0.03}{0.01} = 3 = \frac{r_2}{r_1}$

$\frac{v_2}{v_1} = \left(\frac{r_2}{r_1}\right)^3 = (3)^3 = 27$

29. (A)

The fundamental frequency is given by

$n = \frac{1}{2\ell r}\sqrt{\frac{T}{\pi\rho}}$

$\therefore \quad n \propto \frac{1}{\ell r}$

$\therefore \quad \frac{n_2}{n_1} = \frac{\ell_1 r_1}{\ell_2 r_2} = \frac{1}{2} \times \frac{1}{2} = \frac{1}{4}$

$\therefore \quad n_2 = \frac{n_1}{4} = \frac{n}{4}$

30. (A)

$e = e_0 \sin\omega t$

If $e = \frac{e_0}{2}$ then $\frac{e_0}{2} = e_0 \sin \omega t$

$\therefore \quad \sin\omega t = \frac{1}{2} \qquad \therefore \quad \omega t = 30° = \frac{\pi}{6} \text{ rad}$

$\therefore \quad \frac{2\pi}{T} \cdot t = \frac{\pi}{6} \qquad \therefore \quad t = \frac{T}{12}$

31. (D)

The emergent ray just grazes the second face.

Hence angle of emergence e = 90°

$\mu = \frac{\sin e}{\sin r_2} = \frac{\sin 90°}{\sin r_2} = \frac{1}{\sin r_2}$

$\therefore \quad \frac{1}{\sin r_2} = \sqrt{2} \quad \text{or} \quad \sin r_2 = \frac{1}{\sqrt{2}}$

$\therefore \quad r_2 = 45°; \qquad A = r_1 + r_2 \qquad \therefore \quad r_1 = A - r_2 = 60 - 45 = 15°$

Also, $\frac{\sin i}{\sin r_1} = \mu \quad \therefore \quad \sin i = \mu \sin r_1 = \sqrt{2} \sin 15°$

$\therefore \quad i = \sin^{-1}\left(\sqrt{2} \sin 15°\right)$

32. (B)

In a half–wave rectifier, one pulse is obtained for each wave and hence 60 pulses will be obtained per second.

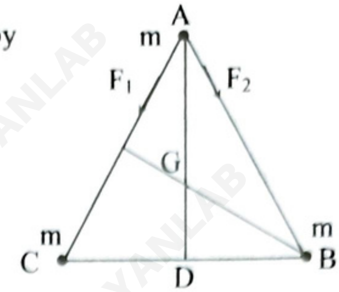
In a full wave rectifier, two pulses are obtained for each wave and 120 pulses are obtained per second.

$\therefore$ (B)

33. (C)

Consider mass m at A.

The forces exerted on it by the other two masses are given by



$$F_1 = G\frac{m^2}{L^2} = F_2$$

The angle between the two forces is 60°.

Hence the resultant force

$$F = \sqrt{F_1^2 + F_1^2 + 2F_1^2 \cos 60^\circ} = \sqrt{3}F_1$$

$$\therefore \ F = \sqrt{3}\cdot G\frac{m^2}{L^2}$$

Mass m rotates around the centre G. The radius of the circular motion is AG.

$$AG = \frac{2}{3}AD;$$

$$AD = AC\sin 60^\circ = L\sin 60^\circ = \frac{\sqrt{3}}{2}L$$

$$\therefore \ AG = \frac{2}{3}\cdot\frac{\sqrt{3}}{2}L = \frac{L}{\sqrt{3}}$$

$$\therefore \ \text{radius } r = \frac{L}{\sqrt{3}}$$

For uniform circular motion, the gravitational force provides the centripetal force.

$$\therefore \ mr\omega^2 = F$$

$$\therefore \ m\frac{L}{\sqrt{3}}\cdot\omega^2 = \sqrt{3}G\frac{m^2}{L^2}$$

$$\therefore \ \omega^2 = 3G\frac{m}{L^3}$$

$$\therefore \ \omega = \left(3G\frac{m}{L^3}\right)^{1/2}$$

$$\therefore \ \frac{2\pi}{T} = \left(\frac{3Gm}{L^3}\right)^{1/2}$$

$$\therefore \ T = 2\pi\left(\frac{L^3}{3Gm}\right)^{1/2}$$

$$\therefore \ T \propto L^{3/2}$$

34. (D)

Fundamenetal frequency of closed pipe $n_c = \frac{v}{4L}$

Fundamental frequency of open pipe $n_0 = \frac{v}{2L}$

They produce 2 beats per second

$$\therefore \ n_o - n_c = 2, \quad \therefore \ \frac{v}{2L} - \frac{v}{4L} = 2$$

$$\therefore \ \frac{v}{4L} = 2 \ \text{ or } \ \frac{v}{L} = 8$$

When length of open pipe is halved

$$n_o' = \frac{v}{2\left(\frac{L}{2}\right)} = \frac{v}{L}$$

When length of closed pipe is doubled

$$n_c' = \frac{v}{4\times 2L} = \frac{v}{8L}$$

New beat frequency $= n_o' - n_c' = \frac{v}{L} - \frac{v}{8L} = \frac{7v}{8L} = \frac{7}{8}\times 8 = 7$

35. (A)

$B = 0.1\ T,\ \ell = 0.12\ m,\ v = 20\ m/s,\ R = 2\ \Omega$

emf induced $e = B\ell v = 0.1\times 0.12\times 20 = 0.24\ V$

$$I = \frac{V}{R} = \frac{0.24}{2} = 0.12\ A$$

36. (A)

$T = 6000\ K,\ b = 2.897\times 10^{-3}\ m\text{–}K$

By Wien's law, $\lambda T = b$

$$= \frac{2.897\times 10^{-3}}{6000}$$

$$= 4.828\times 10^{-7}\ m = 4828\ Å$$

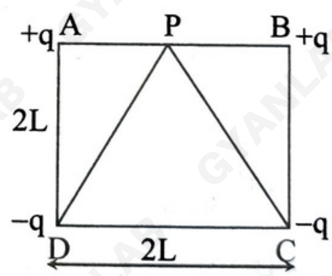
37. (B)

If k is the kinetic energy and p is the momentum then

$$k = \frac{p^2}{2m} \qquad \therefore\ p^2 = 2mk$$

If k is constant, then $p^2 \propto m$

38. (D)



$$DP^2 = (2L)^2 + L^2 = 4L^2 + L^2 = 5L^2$$

$\therefore\ DP = \sqrt{5}L = CP$

$AP = BP = L$

$\therefore$ Potential at point P due to the four charges

$$V = \frac{1}{4\pi\varepsilon_0}\left(\frac{q}{L} + \frac{q}{L} - \frac{q}{\sqrt{5}L} - \frac{q}{\sqrt{5}L}\right)$$

$$= \frac{1}{4\pi\varepsilon_0}\left(\frac{2q}{L} - \frac{2q}{\sqrt{5}L}\right)$$

$$= \frac{1}{4\pi\varepsilon_0}\cdot\frac{2q}{L}\left(1 - \frac{1}{\sqrt{5}}\right)$$

39. (C)

If wavelengths in air are 480 nm and 672 nm, then wavelengths in a medium of refractive index 1.6 will be

$$\frac{480}{1.6} = 300\ nm \ \text{ and } \ \frac{672}{1.6} = 420\ nm$$

40. (B)

When temperature is 80°C, by Newton's law of cooling, we have

$$1.5 = k\left(\frac{80+30}{2} - 30\right) = k(55-30) = 25\,K$$

When temperature is 50°C, let r be the rate of cooling.
Then

$$r = k\left(\frac{50+30}{2} - 30\right) = k(40-30) = 10\,K$$

$$\therefore \frac{r}{1.5} = \frac{10}{25} \quad \therefore r = \frac{10}{25}\times 1.5 = \frac{3}{5} = 0.6°C/\min$$

41. (B)

Let E_1 be the energy of the first energy level

Then $E_2 = \frac{E_1}{4}$

Highest level $E_\infty = 0$

$$\therefore \quad E_2 - E_1 = \frac{E_1}{4} - E_1 = -\frac{3}{4}E_1$$

$$E_\infty - E_2 = 0 - \frac{E_1}{4} = -\frac{E_1}{4}$$

$$\therefore \quad \frac{E_2 - E_1}{E_\infty - E_2} = 3$$

42. (A)

If g is the acceleration due to gravity on earth's surface and g' on the planet, then

$$g = \frac{GM}{r^2} \text{ and } g' = \frac{G\times 1.5M}{(1.5)^2 r^2} = \frac{1}{1.5}\frac{GM}{r^2}$$

$$\therefore g' = \frac{g}{1.5}$$

For second's pendulum T = 2 s

$$\therefore T = 2\pi\sqrt{\frac{\ell}{g}} = 2\pi\sqrt{\frac{\ell'}{g'}}$$

$$\therefore \frac{\ell}{g} = \frac{\ell'}{g'} \quad \therefore \ell' = \ell\cdot\frac{g'}{g} = 1\times\frac{1}{1.5} = \frac{2}{3} = 0.67\,m$$

43. (C)

$B = \mu_0 nI$

$$n = \frac{N}{L} = \frac{4\times 1000}{2} = 2000/m,\ I = 5\,A$$

$$\therefore B = 4\pi\times 10^{-7}\times 2000\times 5$$
$$= 4\pi\times 10^{-3}\,T$$

44. (C)

Angular momentum (L) = $I\omega = mr^2\omega$

Magnetic moment (M) = $\pi r^2 q f = \pi r^2 q \dfrac{\omega}{2\pi} = \dfrac{1}{2} q\omega r^2$

$\therefore \dfrac{M}{L} = \dfrac{q}{2m}$

45. (B)

Energy stored in the capacitor $U = \dfrac{1}{2} qV$

Work done by the battery $W = qV$

$\therefore \dfrac{U}{W} = \dfrac{1}{2}$

46. (B)

Kinetic energy is given by

$(K.E.)_{max} = h\nu - h\nu_0$

Comparing $y = mx + c$

We get x−intercept when

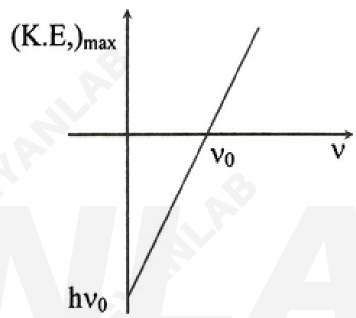
$(K.E.)_{max} = 0$, i.e. $h\nu - h\nu_0 = 0$

or $\nu = \nu_0 = A$

We get y−intercept when $\nu = 0$

$\therefore (K.E.)_{max} = -h\nu_0 = B$

$\therefore \left|\dfrac{B}{A}\right| = \dfrac{h\nu_0}{\nu_0} = h$



47. (B)

If Vis the volume then we have

$V = \dfrac{4}{3}\pi r^3$

$V \propto r^3 \quad \therefore \dfrac{V_2}{V_1} = \left(\dfrac{r_2}{r_1}\right)^3$

$\therefore \dfrac{r_2}{r_1} = \left(\dfrac{v_2}{v_1}\right)^{1/3} = (2)^{\frac{1}{3}}$

$W_1 = 8\pi r_1^2 \cdot T$ and $W_2 = 8\pi r_2^2 T$

$\therefore \dfrac{W_2}{W_1} = \left(\dfrac{r_2}{r_1}\right)^2 = (2)^{\frac{2}{3}} = (4)^{\frac{1}{3}}$

$\therefore W_2 = (4)^{1/3} W_1$

48. (A)

If the number of waves is N, then we wavelength in water is $\frac{5}{N}$ cm and wavelength in glass is $\frac{4}{N}$ cm.

$$\lambda_w = \frac{5}{N}, \lambda_g = \frac{4}{n}$$

$$_w\mu_g = \frac{\lambda_w}{\lambda_g} = \frac{5}{4}$$

$$\therefore \frac{\mu_g}{\mu_w} = \frac{5}{4}$$

$$\therefore \mu_w = \frac{4}{5}\mu_g = \frac{4}{5}\times\frac{5}{3} = \frac{4}{3} = 1.33$$

49. (C)

In series combination, the equivalent capacitance C is given by

$$\frac{1}{C} = \frac{1}{C_1} + \frac{1}{C_2} + \frac{1}{C_3}$$

$$\therefore \frac{1}{C} = \frac{C_2C_3 + C_1C_3 + C_1C_2}{C_1C_2C_3}$$

$$\therefore C = \frac{C_1C_2C_3}{C_2C_3 + C_1C_3 + C_1C_2}$$

Charge Q stored by the combinations is given by

$$Q = CV = \frac{C_1C_2C_3V}{C_2C_3 + C_1C_3 + C_1C_2}$$

Charge on each capacitor is same. Hence potential difference across C_1 is $V_1 = \frac{Q}{C_1}$

$$\therefore V_1 = \frac{C_2C_3V}{C_2C_3 + C_1C_3 + C_1C_2}$$

50. (A)

$$\frac{V_2}{V_1} = \frac{1}{8}, \gamma = \frac{5}{3}$$

For adiabatic process,

$$P_1V_1^{\gamma} = P_2V_2^{\gamma}$$

$$\therefore \frac{P_2}{P_1} = \left(\frac{V_1}{V_2}\right)^{\gamma} = (8)^{5/3} = (2)^5 = 32$$

CHEMISTRY

51. (B)

52. (D)

53. (D)

EAN $= Z - X + Y$

Here, EAN = 36, Z = 27, Y = 12 ($\because$ C.N. = 6)

$\therefore$ Oxidation state (X) of Co = Z + Y − EAN

$= 27 + 12 - 36 \quad = +3$

54. (C)

$V^{3+} : 3d^2 \rightarrow$ 2 unpaired electrons

$Ti^{3+} : 3d^1 \rightarrow$ 1 unpaired electron

$Cu^{+} : 3d^{10} \rightarrow$ No unpaired electron

$Mn^{2+} : 3d^5 \rightarrow$ 5 unpaired electrons

Due to absence of unpaired electron, Cu^+ will not form coloured compounds.

55. (A)

Due to bulky aryl group in benzaldehyde.

56. (B)

57. (B)

In Cannizzaro reaction, one molecule of an aldehyde is reduced to alcohol and at the same time second molecule is oxidized to carboxylic acid salt. This is an example of disproportionation reaction.

58. (D)

$W_2 = 3$ g, $M_2 = 60$ g mol^{-1}

$W_2(A) = 4.5$ g, $M_2(A) = ?$

Two aqueous solutions boils at same temperature,

$$\therefore \frac{M_2}{W_2} = \frac{M_2(A)}{W_2(A)}$$

$$\therefore M_2(A) = \frac{M_2 \times W_2(A)}{W_2}$$

$$= \frac{60\,g\,mol^{-1} \times 4.5\,g}{3g} \qquad = 90\ g\ mol^{-1}$$

59. (D)

60. (A)

$[:\ddot{\underset{..}{S}} - C \equiv N:]^{\ominus}$

$$FC = VE - NE - \frac{BE}{2}$$

$$\therefore \text{ Formal charge of N atom } = 5 - 2 - \frac{6}{2} \quad = 0$$

61. (B)

$$CH_3COOH + CH_3CH_2OH \xrightleftharpoons{H^+} CH_3COOCH_2CH_3 + H_2O \xrightarrow[Ni/Pd,\ \Delta]{H_2} 2CH_3-CH_2-OH$$

Ethanoic acid   Ethanol   Ethyl ethanoate   Ethanol

62. (C)

$S = 1.0 \times 10^{-4}$ mol dm^{-3}, $K_{sp} = ?$

$AB_2 \rightleftharpoons A^{2+} + 2B^-$

Here, x = 1 and y = 2

$\therefore\ K_{sp} = 4S^3$

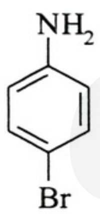
$= 4 \times (1.0 \times 10^{-4})^3 \quad = 4 \times 10^{-12}$

63. (A)

Sc^{3+} : $3d^0$ $\rightarrow$ No unpaired electron
Ti^{3+} : $3d^1$ $\rightarrow$ 1 unpaired electron
V^{3+} : $3d^2$ $\rightarrow$ 2 unpaired electrons
Fe^{3+} : $3d^5$ $\rightarrow$ 5 unpaired electrons

Due to absence of unpaired electron, Sc in +3 oxidation state forms colourless compounds.

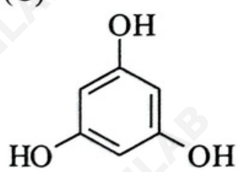
64. (D)



$\rightarrow$ Contains amino ($-NH_2$) group

4–Bromoaniline

65. (C)



Common name : Phloroglucinol

IUPAC name : Benzene-1,3,5-triol

66. (A)

t = 4.606 min, k = 0.1989 min^{-1}

For first order reaction,

$$t_{1/2} = \frac{0.693}{k}$$

$$= \frac{0.693}{0.1989\ \text{min}^{-1}} = 3.48\ \text{min}$$

67. (A)

$\rho = 7\ g\ cm^{-3}$, $a = 300\ pm = 3 \times 10^{-8}\ cm$, $M = 52\ g\ mol^{-1}$

$$M = \rho \frac{a^3 N_A}{n}$$

$$\therefore\ n = \frac{\rho a^3 N_A}{M}$$

$$= \frac{7\,g\,cm^{-3} \times (3\times10^{-8})^3\,cm^3 \times 6.022\times10^{23}\,atoms\ mol^{-1}}{52\,g\,mol^{-1}}$$

$$= \frac{7\times27\times10^{-24}\times6.022\times10^{23}}{52}\ atoms$$

$\therefore\ n = 2.19 \approx 2$ atoms

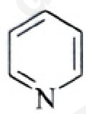
As n = 2, it is body centred cubic unit cell.

68. (C)

1 mol of H_2O = 18 g at STP

$\therefore$ 0.25 mol of $H_2O = \frac{18\times0.25}{1} = 4.5$ g

69. (D)



Pyridine (C_5H_5N)

70. (D)

71. (C)

As acetic acid having lowest molecular mass, it has lowest boiling point.

72. (B)

$$2A + B \longrightarrow 2C$$

$$\text{Rate} = -\frac{1}{2}\frac{d[A]}{dt} = -\frac{d[B]}{dt} = \frac{1}{2}\frac{d[C]}{dt}$$

$$\therefore\ \frac{d[B]}{dt} = \frac{1}{2}\frac{d[A]}{dt}$$

$$= \frac{1}{2}\times0.076\ mol\,s^{-1} = 0.038\ mol\ s^{-1}$$

73. (D)

74. (A)

According to First Law of thermodynamics,

$\Delta U = Q + W$

$\therefore\ \Delta U = Q - P_{ext}\,\Delta V$ $\quad (\because\ W = -P_{ext}\,\Delta V)$

But for isochoric process,

$\Delta V = 0$

$\therefore\ \Delta U = Q_V$

75. (B)
As branching increases, boiling point decreases.
$\therefore$ tert-butyl alcohol has lowest boiling point.

76. (D)

$$H_2SO_4 \longrightarrow 2H^+ + SO_4^{2-}$$
$$0.005\ M \quad 2 \times 0.005 = 0.01\ M \quad 0.005\ M$$

$$pH = -\log_{10}[H^+] = -\log_{10}[10^{-2}] = 2.0$$

77. (B)

$$Ca_3(PO_4)_2 \rightleftharpoons 3Ca^{2+} + 2PO_4^{3-}$$
$$\therefore K_{sp} = [Ca^{2+}]^3 [PO_4^{3-}]^2$$

78. (A)
In S_N1 mechanism, nucleophile can attack planar carbocation from either side.

79. (A)
a = 600 pm, r = ?
For BCC structure,

$$r = \frac{\sqrt{3}}{4}a = \frac{\sqrt{3}}{4} \times 600\,pm = \sqrt{3} \times 150\,pm$$

80. (A)
$P_{ext} = 2.40 \times 10^5$ Pa, $V_2 = 2.2 \times 10^{-3}$ m^3
W = –0.048 kJ = –48 J, V_1 = ?

$$W = -P_{ext}(V_2 - V_1)$$
$$\therefore -48 = -2.40 \times 10^5 (2.2 \times 10^{-3} - V_1)$$
$$\therefore 48 = 5.28 \times 10^2 - (2.40 \times 10^5 V_1)$$
$$\therefore (2.40 \times 10^5) V_1 = 528 - 48$$
$$\therefore V_1 = \frac{480}{2.40 \times 10^5} = 200 \times 10^{-5}$$
$$\therefore V_1 = 2.0 \times 10^{-3}\ m^3$$

81. (C)

82. (C)
V_1 = 250 mL, P_1 = 2 atm
V_2 = ?, P_2 = 2.5 atm
According to Boyle's law,

$$P_1V_1 = P_2V_2$$
$$\therefore V_2 = \frac{P_1V_1}{P_2}$$
$$\therefore = \frac{2\ atm \times 250\ mL}{2.5\ atm} = 200\ mL$$

83. (B)

$$\frac{P_1^0 - P_1}{P_1^0} = x_2$$

$$\therefore \frac{P_1^0}{P_1^0} - \frac{P_1}{P_1^0} = x_2$$

$$\therefore x_2 = 1 - 0.15 = 0.85$$

84. (D)

85. (D)

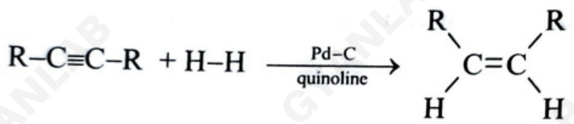
W = 62 J, Q = –128 J

According to first law of thermodynamics,

$$\Delta U = Q + W$$

$$= -128 + 62 = -66 \text{ J}$$

86. (B)



Alkyne cis-Alkene

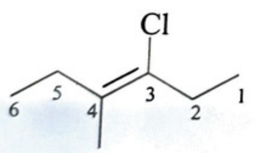
87. (B)

$$BaO_{2\,(g)} + O_{3\,(g)} \rightarrow BaO_{(s)} + 2O_{2\,(g)}$$

Ozone reduces peroxide to oxide.

88. (B)

89. (D)



3-Chloro-4-methylhex-3-ene

90. (C)

Chlorous acid : $H\overset{+3}{Cl}O_2$

Chloric acid : $H\overset{+5}{Cl}O_3$

Perchloric acid : $H\overset{+7}{Cl}O_4$

Hypochlorous acid : $HO\overset{+1}{Cl}$

91. (A)

92. (A)

ℓ = 2 for d-orbital which accommodate 10 electrons.

93. (C)

94. (D)

$c = 0.02$ M, $\wedge = 412.3\ \Omega^{-1}\ cm^2\ mol^{-1}$

$$\wedge = \frac{1000\,k}{c}$$

$$\therefore\ k = \frac{\wedge c}{1000}$$

$$= \frac{412.3\Omega^{-1}\,cm^2\,mol^{-1} \times 0.02\,mol\,L^{-1}}{1000\ cm^3\ L^{-1}}$$

$$= 8.246 \times 10^{-3}\ \Omega^{-1}\ cm^{-1}$$

95. (C)

96. (B)

$$\%\ \text{atom economy} = \frac{\text{Formula weight of the product}}{\text{Formula weight of the reactant}} \times 100$$

$$\therefore\ \%\ \text{atom economy} = \frac{123}{246} \times 100 \qquad = 50\%$$

97. (D)

The electrode reaction is

$$Cu^{2+} + 2e^- \longrightarrow Cu$$

$$\therefore\ 2Cu^{2+} + 4e^- \longrightarrow 2Cu$$

The quantity of charge required for reduction of 2 moles of Cu^{2+} = 4 F

= 4 × 96500 C

= 3.86×10^5 C

98. (D)

Aryl amines are weaker bases than aliphatic amines.

99. (C)

$$H_2\overset{\overset{6+}{\uparrow}}{S}O_4 + \overset{\overset{2+}{\uparrow}}{Ca}(OH)_2 \longrightarrow \overset{\overset{2+}{\uparrow}}{Ca}\overset{\overset{6+}{\uparrow}}{S}O_4 + H_2O$$

No change in oxidation number of any species, hence it is not a redox reaction.

100. (D)

Section II

MATHEMATICS

101.(C)

$P(E_1 \cup E_2) = P(E_1) + P(E_2) - P(E_1 \cap E_2)$

$\therefore\ 0.6 = [1 - P(E_1')] + [1 - P(E_2')] - 0.2$

$\therefore\ P(E_1') + P(E_2') = 2 - 0.2 - 0.6 = 1.2$

102.(A)

$$\lim_{x\to 0}\frac{\sqrt{1-\cos x^2}}{1-\cos x}$$

$$= \lim_{x\to 0}\frac{\sqrt{2\sin^2\frac{x^2}{2}}}{2\sin^2\frac{x}{2}} = \lim_{x\to 0}\frac{\sqrt{2}\sin\frac{x^2}{2}}{2\sin^2\frac{x}{2}}$$

Dividing numerator and denominator by $\frac{x^2}{4}$, we get

$$= \frac{\frac{1}{\sqrt{2}}\lim_{x\to 0}\left[\frac{\sin\left(\frac{x^2}{2}\right)}{\left(\frac{x^2}{4}\right)}\right]}{\frac{\sin\left(\frac{x}{2}\right)}{\left(\frac{x}{2}\right)}\times\frac{\sin\left(\frac{x}{2}\right)}{\left(\frac{x}{2}\right)}} = \frac{\frac{1}{\sqrt{2}}\lim_{x\to 0}\frac{\sin\left(\frac{x^2}{2}\right)}{\left(\frac{x^2}{2}\right)\times\frac{1}{2}}}{\lim_{x\to 0}\left[\frac{\sin\left(\frac{x}{2}\right)}{\left(\frac{x}{2}\right)}\right]^2} = \frac{1}{\sqrt{2}}\times\frac{2}{1} = \sqrt{2}$$

103.(B)

$$A = \begin{bmatrix}\lambda & i\\ i & -\lambda\end{bmatrix} \Rightarrow |A| = \begin{vmatrix}\lambda & i\\ i & -\lambda\end{vmatrix}$$

Since A^{-1} does not exist, we write $|A| = 0$

$\therefore\ (-\lambda^2) - (i^2) = 0 \Rightarrow \lambda^2 = 1 \Rightarrow \lambda = \pm 1.$

104.(C)

$s = 2 + 27t - t^3$ and particle stops when its velocity is zero.

$\therefore\ \frac{ds}{dt} = 27 - 3t^2 = 0 \Rightarrow t^2 = 9 \Rightarrow t = 3$ sec.

$\therefore$ Distance covered in 3 sec, is

$s_{(t=3)} = 2 + 27(3) - (3)^3 = 56$ m

105.(A)

Let the Cartesian coordinates by (x, y)

We have $\sqrt{x^2+y^2} = 2 \Rightarrow x^2 + y^2 = 4$ and $\tan\frac{\pi}{4} = \frac{y}{x} \Rightarrow 1 = \frac{y}{x} \Rightarrow x = y.$

$\therefore\ 2x^2 = 4 \Rightarrow x^2 = 2 \Rightarrow x = \pm\sqrt{2} \Rightarrow y = \pm\sqrt{2}$

106.(A)

Let $I = \int e^x\left(\frac{1+\sin x}{1+\cos x}\right)dx = \int e^x\left(\frac{1}{1+\cos x}+\frac{\sin x}{1+\cos x}\right)dx$

$= \int e^x\left(\frac{1}{2\cos^2\frac{x}{2}}+\frac{2\sin\frac{x}{2}\cos\frac{x}{2}}{2\cos^2\frac{x}{2}}\right)dx = \int e^x\left(\frac{1}{2}\right)\left(\sec^2\frac{x}{2}+2\tan\frac{x}{2}\right)dx$

$= \frac{1}{2}\int e^x\left[2\tan\frac{x}{2}+\sec^2\frac{x}{2}\right]dx = \frac{1}{2}\cdot e^x(2)+\tan\left(\frac{x}{2}\right)+c = e^x\tan\frac{x}{2}+c$

107.(D)

Let p : $\forall x \in N$ and q : $x^2 + x$ is even number.

The logical form of given statement is $p \wedge q$.

$\sim(p \wedge q) \equiv \sim p \vee \sim q$ i.e. $\exists x \in N$ such that $x^2 + x$ is not an even number.

108.(A)

Let $I = \int_2^5 2[x]dx$

$= \int_2^3 2(2)dx + \int_3^4 2(3)dx + \int_4^5 2(4)dx$

$= 4[x]_2^3 + 6[x]_3^4 + 8[x]_4^5 = 4 + 6 + 8 = 18$

109.(C)

Let $I = \int_0^\pi x\sin x\cos^4 x\,dx$ ...(1)

$= \int_0^\pi (\pi - x)\sin(\pi - x)[\cos(\pi - x)]^4 dx$

$= \int_0^\pi (\pi - x)\sin x[\cos x]^4 dx$ ...(2)

Eq. (1) + (2) gives,

$2I = \int_0^\pi \pi\sin x\cos^4 x\,dx$

Put $\cos x = t \Rightarrow -\sin x\,dx = dt$

When $x = 0$, $t = 1$ and when $x = \pi$, $t = -1$

$2I = \pi\int_1^{-1}(t)^4(-dt)$

$= \pi\int_{-1}^1 t^4 dt = 2\pi\int_0^1 t^4 dt$ ... [t^4 is an even function]

$\therefore 2I = \frac{2\pi}{5}\left[t^5\right]_0^1 \Rightarrow I = \frac{\pi}{5}$

110.(A)

The required plane passes through (2, −3, 1).

It is normal to the line having d.r.s. (1, 5, −6).

$\therefore\ x + 5y - 6z = k \quad \Rightarrow 2 + 5(-3) - 6(1) = k$ i.e. $k = -19$

Hence equation of plane is $x + 5y - 6z + 19 = 0$

111.(B)

The committee can be formed in following ways :

(5 men), (4 men, 1 lady), (3 men, 2 ladies).

$\therefore$ Number of ways. $= \left({}^6C_5\right) + \left({}^6C_4 \times {}^4C_1\right) + \left({}^6C_3 \times {}^4C_2\right)$

$= (6) + (15 \times 4) + (20 \times 6) \quad = 6 + 60 + 120 = 186$

112.(A)

$$\frac{\cos 2A}{a^2} - \frac{\cos 2B}{b^2}$$

$$= \frac{1 - 2\sin^2 A}{a^2} - \frac{1 - 2\sin^2 B}{b^2} \quad = \left(\frac{1}{a^2} - \frac{1}{b^2}\right) - 2\left(\frac{\sin^2 A}{a^2} - \frac{\sin^2 B}{b^2}\right)$$

From sine rule, we know that

$$\frac{a}{\sin A} = \frac{b}{\sin B}$$

$$\therefore \text{ Given Expression } = \left(\frac{1}{a^2} - \frac{1}{b^2}\right) - 0 = \frac{1}{2^2} - \frac{1}{3^2} \quad = \frac{1}{4} - \frac{1}{9} = \frac{5}{36}$$

113.(B)

$(1 + e^{2x})\, dy + e^x (1 + y^2)\, dx = 0$

$$\therefore \frac{dy}{1 + y^2} + \frac{e^x}{(1 + e^{2x})} dx \quad = 0$$

$$\therefore \int \frac{dy}{1 + y^2} \quad = -\int \frac{e^x}{1 + e^{2x}} dx$$

Put $e^x = t \ \Rightarrow \ e^x\, dx = dt$

$$\therefore \int \frac{dy}{1 + y^2} \quad = -\int \frac{dt}{1 + t^2} \quad \Rightarrow \tan^{-1}(y) \quad = -\tan^{-1}(t) + c$$

$\therefore\ \tan^{-1}(y) + \tan^{-1}(e^x) = c$

We have $x = 0,\ y = 1$

$$\therefore \tan^{-1}(1) + \tan^{-1}(e^{\circ}) = c \quad \Rightarrow c = 2\tan^{-1}(1) = \frac{\pi}{2}$$

$$\therefore \tan^{-1} y + \tan^{-1} e^x \quad = \frac{\pi}{2}$$

114.(A)

$y \ = 1 + x e^y$

$$\therefore \frac{dy}{dx} \quad = 0 + xe^y \frac{dy}{dx} + e^y$$

$$\therefore \frac{dy}{dx}\left(xe^y - 1\right) \quad = -e^y \ \Rightarrow \frac{dy}{dx} = \frac{-e^y}{xe^y - 1}$$

$$\therefore \frac{dy}{dx} \quad = \frac{-e^y}{\left(1 + xe^y\right) - 2} \quad = \frac{-e^y}{y - 2} \quad = \frac{e^y}{2 - y}$$

115.(C)

Probability of getting 6 = $\frac{1}{6}$

X denotes number of times of getting 6. So x can take values 0, 1, 2.

$$P(x=0) = \frac{5}{6}\times\frac{5}{6} = \frac{25}{36}$$

$$P(x=1) = \left(\frac{1}{6}\times\frac{5}{6}\right)+\left(\frac{5}{6}\times\frac{1}{6}\right) = \frac{10}{36}$$

$$P(x=2) = \frac{1}{6}\times\frac{1}{6} = \frac{1}{36}$$

$$E(x) = \Sigma p_i x_i$$

$$= \left(\frac{25}{36}\right)(0)+\left(\frac{10}{36}\right)(1)+\left(\frac{1}{36}\right)(2) = \frac{10+2}{36} = \frac{1}{3}$$

116.(A)

A(7, −8, 1); B(p, q, 5) and C(q + 1, 5p, 0) are vertices of ΔABC having centroid G(3, −5, r)

$$\therefore\ 3 = \frac{7+p+q+1}{3},\quad -5 = \frac{-8+q+5p}{3}, r = \frac{1+5+0}{3}$$

$\therefore\ p+q=1 \quad \ldots(1),\ 5p+q=-7 \quad \ldots(2),\ r=2$

Solving (1) and (2), we get $p=-2, q=3$

117.(C)

We have $(x^2+y^2)\sin\theta+2xy=0$

$\therefore\ a=\sin\theta, b=\sin\theta$ and $h=1$ and let α be the acute angle between the given lines.

$$\text{Then } \tan\alpha = \frac{\left|2\sqrt{(1)^2-(\sin\theta)(\sin\theta)}\right|}{\sin\theta+\sin\theta}$$

$$= \frac{\left|2\sqrt{1-\sin^2\theta}\right|}{2\sin\theta} = \frac{2\cos\theta}{2\sin\theta} = \cot\theta = \tan\left(\frac{\pi}{2}-\theta\right)$$

$$\therefore\ \alpha = \frac{\pi}{2}-\theta$$

118.(D)

$$\bar{a}\cdot\left[\left(\bar{b}\times\bar{c}\right)\times\left(\bar{a}+\bar{b}+\bar{c}\right)\right]$$

$$=\bar{a}\cdot\left[\left(\bar{b}\times\bar{a}\right)+\left(\bar{b}\times\bar{b}\right)+\left(\bar{b}\times\bar{c}\right)+\left(\bar{c}\times\bar{a}\right)+\left(\bar{c}\times\bar{b}\right)+\left(\bar{c}\times\bar{c}\right)\right]$$

$$=\bar{a}\cdot\left(\bar{b}\times\bar{a}\right)+\bar{a}\cdot(0)+\bar{a}\cdot\left(\bar{b}\times\bar{c}\right)+\bar{a}\cdot\left(\bar{c}\times\bar{a}\right)+\bar{a}\cdot\left(\bar{c}\times\bar{b}\right)+\bar{a}\cdot(0)$$

$$=0+\bar{a}\cdot\left(\bar{b}\times\bar{c}\right)+0-\bar{a}\cdot\left(\bar{b}\times\bar{c}\right)=0$$

119.(C)

Lines $\frac{2(x-2)}{\lambda} = \frac{y-1}{2} = \frac{z-3}{1}$ and $\frac{x-1}{1} = \frac{3\left(y-\frac{1}{3}\right)}{\lambda} = \frac{z-2}{1}$

are perpendicular to one another.

$\therefore \left(\frac{\lambda}{2}\right)(1)+(2)\left(\frac{\lambda}{3}\right)+(1)(1)=0$

$\therefore \frac{\lambda}{2}+\frac{2\lambda}{3}=-1 \Rightarrow \lambda=\frac{-6}{7}$

120.(B)

$\sqrt{\frac{dy}{dx}}-4\frac{dy}{dx}-7x=0$

$\therefore \sqrt{\frac{dy}{dx}}=4\frac{dy}{dx}+7x$ and squaring both sides, we get

$\frac{dy}{dx}=16\left(\frac{dy}{dx}\right)^2+56x\left(\frac{dy}{dx}\right)+49x^2$

$\therefore$ Order = 1, degree = 2

121.(B)

$x=e^t(\sin t-\cos t)$ and $y=e^t(\sin t+\cos t)$

$\therefore \frac{dx}{dt}=e^t(\sin t-\cos t)+e^t(\cos t+\sin t)=2e^t\sin t$

$\frac{dy}{dt}=e^t(\sin t+\cos t)+e^t(\cos t-\sin t)=2e^t\cos t$

$\therefore \frac{dy}{dx}=\frac{2e^t\cos t}{2e^t\sin t} \Rightarrow \left(\frac{dy}{dx}\right)_{t=\frac{\pi}{3}}=\cot\left(\frac{\pi}{3}\right)=\frac{1}{\sqrt{3}}$

122.(D)

The lines $(k^2+2)x^2+3xy-6y^2=0$ are perpendicular to each other.

$\therefore (k^2+2)+(-6)=0 \Rightarrow k^2=4 \Rightarrow k=\pm 2$

123.(B)

We have $\bar{a}=3\hat{i}+\hat{j}-\hat{k}$, $\bar{b}=2\hat{i}-\hat{j}+23\hat{k}$ and $\bar{c}=7\hat{i}-\hat{j}+23\hat{k}$

$\bar{a}\cdot\bar{b}=6-1-23\neq 0$ and $\bar{b}\cdot\bar{c}=14+1+529\neq 0$

Thus $\bar{a},\bar{b},\bar{c}$ are not mutually perpendicular.

Also for $\bar{a}$ and $\bar{b}$, $\frac{3}{2}\neq -1\neq \frac{-1}{23}$.

Thus $\bar{a}$ and $\bar{b}$ are not collinear.

Now $\begin{vmatrix}3 & 1 & -1\\ 2 & -1 & 23\\ 7 & -1 & 23\end{vmatrix}=3(-23+23)-(46-161)-(-2+7)\neq 0$.

Thus $\bar{a},\bar{b},\bar{c}$ are non coplanar.

124.(A)

We have $n=50$, $\Sigma x_i^2=3050$, $\bar{x}=6$

S.D. $=\sqrt{\frac{1}{n}\Sigma x_i^2-(\bar{x})^2}$

$=\sqrt{\frac{3050}{50}-(6)^2}=\sqrt{61-36}=\sqrt{25}=5$

125.(A)

$x^2 - 10x + y^2 = 0$

$x - 10x + 25 + y^2 = 25 \quad \Rightarrow$ centre = (5, 0) and r = 5

y = 2x is a chord of given circle.

Point of intersection of chord and circle is

$x^2 - 10x + 25 + 4x^2 = 25 \quad \Rightarrow \quad 5x^2 - 10x = 0 \quad \Rightarrow \quad x^2 - 2x = 0$

i.e. $x(x-2) = 0 \Rightarrow \quad x = 0, 2 \Rightarrow \quad y = 0, 4$

Thus end points of the chord are (0, 0) and (2, 4)

Mid point of the chord = $\left(\frac{2}{2}, \frac{4}{2}\right) = (1, 2)$ and

length of chord = $\sqrt{(2)^2 + (4)^2} = \sqrt{20}$ is the diameter of required circle.

Hence equation of required circle is $(x - 1) + (y - 2) = \left(\frac{\sqrt{20}}{2}\right)^2$

i.e. $x^2 + y^2 - 2x - 4y = 0$

126.(C)

$\sin^2 x + \cos^2 y \;= 1$

$\therefore \;\; 2 \sin x \cos x - 2 \cos y \sin y \frac{dy}{dx} \;= 0$

$\therefore \;\; \frac{dy}{dx} \;= \frac{2 \sin x \cos x}{2 \sin y \cos y} \;= \frac{\sin 2x}{\sin 2y}$

127.(C)

We know that $x = [x] + \{x\}$

When $x = -1.4$, we get $[x] = -2$ and $\{x\} = 0.6$

We have $f(x) = 2\{x\} + 5x$

$\therefore \;\; f(-1.4) = 2(0.6) + 5(-1.4) = -7 + 1.2 \;= -5.8$

128.(D)

The curve $y = ax^3 + bx^2 + cx + 5$, touches X axis at (–2, 0)

$\therefore \;\; 0 \;= a(-2)^3 + b(-2)^2 + c(-2) + 5$

$\therefore \;\; 8a - 4b + 2c \;= 5 \qquad \ldots(1)$

$\frac{dy}{dx} = 3ax^2 + 2bx + c$ and at point Q on Y axis, we have $\frac{dy}{dx} = 3$.

Let $Q \equiv (0, k)$

$\therefore \;\; 3 \;= 3a(0) + 2(0) + c \quad \Rightarrow c = 3 \qquad \ldots(2)$

Thus equation (1) becomes $8a - 4b + 6 = 5$ i.e. $8a - 4b = -1 \quad \Rightarrow 2a - b = \frac{-1}{4}$

We will go by options.

(A) When $a = \frac{1}{2}, b = \frac{3}{4}$, we get $2\left(\frac{1}{2}\right) - \frac{3}{4} \neq \frac{-1}{4}$

(D) When $a = \frac{-1}{2}, b = \frac{-3}{4}$, we get $2\left(\frac{-1}{2}\right) + \frac{3}{4} = \frac{-1}{4}$

129.(B)

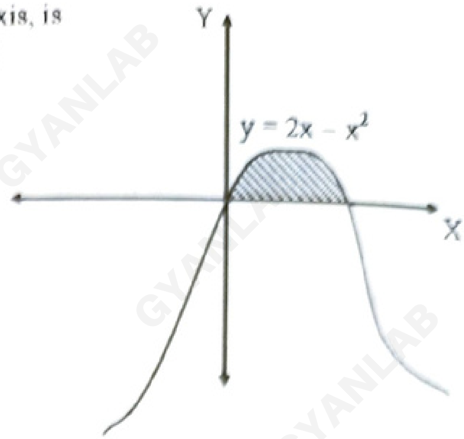
Point of intersection of curve $y = 2x - x^2$ and x axis, is

$0 = 2x - x^2 \Rightarrow x(x-2) = 0 \Rightarrow x = 0, 2$

When $x = 0, y = 0$ and when $x = 2, y = 0$.

Refer figure

Required area is shaded.



$$A = \int_0^2 \left(2x - x^2\right) dx$$

$$= 2\left[\frac{x^2}{2}\right]_0^2 - \left[\frac{x^3}{3}\right]_0^2$$

$$= (4) - \left(\frac{8}{3}\right) = \frac{4}{3} \text{ sq units.}$$

130.(C)

The line passes through point $(0, -2\sqrt{2})$ and has slope $= \frac{-1}{\sqrt{2}}$.

Hence equation of line is,

$$\left(y + 2\sqrt{2}\right) = \frac{-1}{\sqrt{2}}(x - 0) \Rightarrow x + \sqrt{2}\,y + 4 = 0$$

131.(C)

$$\frac{3+2i}{1+i} = \frac{1}{2}(x + iy)$$

$$\therefore\ x + iy = \frac{2(3+2i)}{1+i} \times \frac{1-i}{1-i} = \frac{(6+4i)(1-i)}{1-i^2} = \frac{6+4i-6i-4i^2}{1+1} = 5 - i$$

$\therefore\ x = 5, y = -1 \Rightarrow x - y = 6$

132.(B)

We have $\bar{a} = 2\lambda^2\,\hat{i} + 4\lambda\,\hat{j} + \hat{k}$ and $\bar{b} = 7\hat{i} - 2\hat{j} + \lambda\hat{k}$

$\bar{a}\cdot\bar{b} = |\bar{a}|\cdot|\bar{b}|\cdot\cos\theta$, where θ is angle between $\bar{a}$ and $\bar{b}$.

$\therefore\ \cos\theta = \dfrac{\bar{a}\cdot\bar{b}}{|\bar{a}|\cdot|\bar{b}|} < 0$, as θ is an obtuse angle.

$\therefore\ |\bar{a}|\cdot|\bar{b}| < 0$

$\therefore\ (2\lambda^2)(7) + (4\lambda)(-2) + (1)(\lambda) < 0$

$\therefore\ 14\lambda^2 - 7\lambda < 0 \Rightarrow 7\lambda(2\lambda - 1) < 0 \Rightarrow \lambda(2\lambda - 1) < 0$

$\therefore\ 0 < \lambda < \frac{1}{2}$ i.e. $\lambda \in \left(0, \frac{1}{2}\right)$

133.(A)

$$\frac{dP}{dt} = 0.05\,P$$

$$\therefore\ \int \frac{dP}{0.05p} = \int dt \Rightarrow 20\int \frac{dP}{P} = \int dt$$

$\therefore\ 20 \log |P| = t + c$

When t = 0, P = P $\Rightarrow$ c = 20 log |P|

$\therefore$ 20 log |P| = t + 20 log |P|

When population doubles, we write

20 log |2P| = t + 20 log |P|

$$\therefore\ t = 20\log|2P| - 20\log|P| = 20[\log|2P| - \log|P|] = 20\left[\log\left|\frac{2P}{P}\right|\right]$$

$$= 20(\log 2) \text{ years}$$

134.(A)

$$|A| = \begin{vmatrix} 1 & 2 & 3 \\ -1 & 1 & 2 \\ 1 & 2 & 4 \end{vmatrix} = 1(0) - 2(-6) + 3(-3) = 3$$

We know that A(adj A) = |A| I

$\therefore$ A(adj A) = 3I $\Rightarrow$ k = 3

$(k+1)^4 = (3+1)^4 = 256$

135.(B)

$$\text{Let}\quad I = \int \cos^3 x\, e^{\log(\sin x)^2}\, dx$$

$$= \int \cos^3 x (\sin x)^2\, dx = \int \cos^2 x (\sin x)^2 \cos x\, dx$$

$$= \int (1 - \sin^2 x)(\sin^2 x) \cos x\, dx = \int (\sin^2 x - \sin^4 x) \cos x\, dx$$

Put sin x = t $\Rightarrow$ cos x dx = dt.

$$\therefore\ I = \int (t^2 - t^4)\, dt = \frac{t^3}{3} - \frac{t^5}{5} + c = \frac{\sin^3 x}{3} - \frac{\sin^5 x}{5} + c$$

136.(A)

$$\therefore\ \Sigma p_i x_i = \frac{1}{n} + \frac{2}{n} + \frac{3}{n} + \ldots + \frac{n}{n}$$

$$= \frac{1+2+3+\ldots n}{n} = \frac{n(n+1)}{2(n)} = \frac{n+1}{2}$$

$$\Sigma p_i x_i^2 = \frac{1}{n} + \frac{4}{n} + \frac{9}{n} + \ldots + \frac{n^2}{n}$$

$$= \frac{1+4+9+\ldots+n^2}{n}$$

$$= \frac{n(n+1)(2n+1)}{6n} = \frac{(n+1)(2n+1)}{6}$$

x_i	p_i	$p_i x_i$	$p_i x_i^2$
1	$\frac{1}{n}$	$\frac{1}{n}$	$\frac{1}{n}$
2	$\frac{1}{n}$	$\frac{2}{n}$	$\frac{4}{n}$
3	$\frac{1}{n}$	$\frac{3}{n}$	$\frac{9}{n}$
⋮	⋮	⋮	⋮
n	$\frac{1}{n}$	$\frac{n}{n}$	$\frac{n^2}{n}$

$$\therefore\ \text{Var}(x) = \Sigma p_i x_i^2 - (\Sigma p_i x_i)^2$$

$$= \frac{(n+1)(2n+1)}{6} - \left[\frac{(n+1)}{2}\right]^2 = \frac{2n^2+3n+1}{6} - \frac{n^2+2n+1}{4}$$

$$= \frac{4n^2+6n+2-3n^2-6n-3}{12} = \frac{n^2-1}{12}$$

137.(D)

p : It is raining and q : Weather is plesant.

The symbolic form of given statement is $\sim(p \to \sim q)$.

$\equiv \sim(\sim p \vee \sim q)$

$\equiv p \wedge q$ i.e. It is raining and the weather is pleasant.

138.(C)

$f(x) = x, \quad \text{if } x \le 0$

$\quad = 0, \quad \text{if } x > 0$

$\therefore \lim_{x \to 0^-} f(x) = \lim_{n \to 0} x = 0$ and $\lim_{x \to 0^+} f(x) = 0$

$f(0) = 0$

Thus f(x) is continuous at x = 0.

$f'(x) = 1, \quad \text{if } x \le 0$

$\quad = 0, \quad \text{if } x > 0$

Thus f(x) is not differentiable at x = 0.

139.(B)

p = probability of getting head = $\frac{1}{2}$ and q = probability of not getting head = $\frac{1}{2}$

$$P(X = x) = {}^nC_x \left(\frac{1}{2}\right)^x \left(\frac{1}{2}\right)^{n-x}$$

As per data given, we write

$P(x = 7) = P(x = 9)$

$$\therefore {}^nC_7 \left(\frac{1}{2}\right)^7 \left(\frac{1}{2}\right)^{n-7} = {}^nC_9 \left(\frac{1}{2}\right)^9 \left(\frac{1}{2}\right)^{n-9}$$

$$\therefore \frac{n!}{7!(n-7)!} \times \left(\frac{1}{2}\right)^7 \times \left(\frac{1}{2}\right)^{n-7} = \frac{n!}{9!(n-9)!}\left(\frac{1}{2}\right)^9 \left(\frac{1}{2}\right)^{n-9}$$

$$\frac{(n-9)!(9!)}{(n-7)!\,7!} = 1 \Rightarrow \frac{9 \times 8}{(n-7)(n-8)} = 1$$

$\therefore (n-7)(n-8) = (9)(8) \Rightarrow n - 7 = 9 \Rightarrow n = 16$

when x = 2, we get

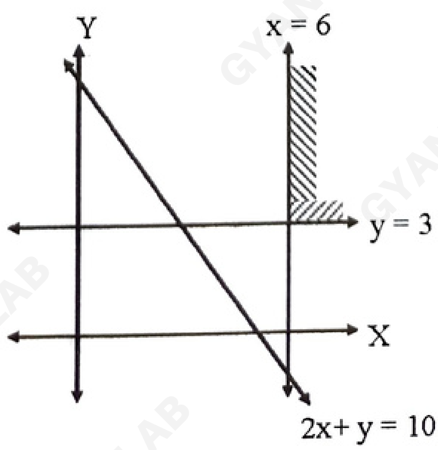
$$P(x = 2) = {}^{16}C_2 \left(\frac{1}{2}\right)^2 \left(\frac{1}{2}\right)^{14} = \frac{16 \times 15}{2}\left(\frac{1}{2}\right)^{16} = \frac{15}{(2)^{13}}$$

140.(B)

Refer figure.

Required area is shaded.

Area is unbounded.



141.(A)

$$\begin{bmatrix} 1 & -1 & 1 \\ 2 & 1 & -3 \\ 1 & 1 & 1 \end{bmatrix}\begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 4 \\ 0 \\ 2 \end{bmatrix}$$

$R_3 \to R_3 - R_1$ and $R_2 \to R_2 - 2R_1$

$$\begin{bmatrix} 1 & -1 & 1 \\ 0 & 3 & -5 \\ 0 & 2 & 0 \end{bmatrix}\begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 4 \\ -8 \\ -2 \end{bmatrix}$$

$\therefore\ x - y + z = 4$ ...(1)

$3y - 5z = -8$ ...(2)

$2y = -2$ ...(3)

$\therefore\ y = -1$ ...[from (3)]

$\therefore\ -3 - 5z = -8 \Rightarrow z = 1$ ...[from (2)]

$\therefore\ x - (-1) + 1 = 4 \Rightarrow x = 2$ ...[from (1)]

$\therefore\ 2x + y - z = 2(2) - 1 - 1 = 2$

142.(B)

Let $I = \int \frac{dx}{e^x + e^{-x} + 2}$

$$= \int \frac{dx}{e^x + \frac{1}{e^x} + 2} = \int \frac{e^x dx}{e^{2x} + 2e^x + 1} = \int \frac{e^x}{\left(e^x + 1\right)^2} dx$$

Put $e^x + 1 = t \Rightarrow e^x\, dx = dt$

$$I = \int \frac{dt}{t^2} = \int t^{-2} dt = \frac{t^{-1}}{-1} + c = \frac{-1}{t} + c = \frac{-1}{e^x + 1} + c$$

143.(A)

$\frac{x+2}{1} = \frac{y-1}{2} = \frac{z+1}{-2} = \lambda$ ...(say)

Hence coordinates of any point on the given line are $(\lambda - 2, 2\lambda + 1, -2\lambda - 1)$.

This point is at a distance of 12 units from $(-2, 1, -1)$.

$$\therefore\ 12 = \sqrt{(\lambda - 2 + 2)^2 + (2\lambda + 1 - 1)^2 + (-2\lambda - 1 + 1)^2} = \sqrt{\lambda^2 + 4\lambda^2 + 4\lambda^2} = 3\lambda$$

$\therefore\ \lambda = \pm 4 \Rightarrow$ Required point $= (2, 9, -9)$ or $(-6, -7, 7)$

144.(A)

$\tan^{-1} 2 + \tan^{-1} 3$

$$= \tan^{-1}\left[\frac{2+3}{1-(2)(3)}\right] = \tan^{-1}\left(\frac{5}{1-6}\right) = \tan^{-1}(-1) = \left(\frac{3\pi}{4}\right)^c$$

145.(D)

The volume of required parallelepiped

$$= (\bar{a} + \bar{b}) \cdot [(\bar{b} + \bar{c}) \times (\bar{c} + \bar{a})]$$

$$= (\bar{a} + \bar{b}) \cdot [(\bar{b} \times \bar{c}) + (\bar{b} \times \bar{a}) + (\bar{c} \times \bar{c}) + (\bar{c} \times \bar{a})]$$

$= [\bar{a}\cdot(\bar{b}\times\bar{c})]+[\bar{a}\cdot(\bar{b}\times\bar{a})]+0+[\bar{a}\times(\bar{c}\times\bar{a})] + [\bar{b}\cdot(\bar{b}\times\bar{c})]+[\bar{b}\cdot(\bar{b}\times\bar{a})]+0+[\bar{b}\cdot(\bar{c}\times\bar{a})]$

$= [\bar{a}\cdot(\bar{b}\times\bar{c})]+0+0+0+0+0+0+[\bar{b}\cdot(\bar{c}\times\bar{a})]$

$= 2\bar{a}\cdot(\bar{b}\times\bar{c})$

146.(C)

Differential Equation of parabolas whose axis is Y axis, is

Let the vertex be (0, k).

$(x-0)^2 = 4b(y-k)$ ...(1)

$\therefore\ x^2 = 4by - 4bk$

$\therefore\ 2x = 4b\dfrac{dy}{dx} - 0 \Rightarrow b = \left(\dfrac{x}{2}\right)\times\dfrac{1}{\left(\dfrac{dy}{dx}\right)}$

Substituting value of b in eq. (1), we get

$$x^2 = 4\left(\frac{x}{2}\right)\times\frac{1}{\left(\frac{dy}{dx}\right)}(y-k)$$

$$\therefore\ x^2\left(\frac{dy}{dx}\right) = 2x(y-k)$$

$$\therefore\ x^2\left(\frac{d^2y}{dx^2}\right)+\left(\frac{dy}{dx}\right)(2x) = 2x\frac{dy}{dx}+2(y-k)$$

$$\therefore\ y-k = \frac{x^2\left(\frac{d^2y}{dx^2}\right)}{2}$$

Substituting value of (y – k) in eq. (1), we get

$$x^2 = 4\left(\frac{x}{2}\right)\left[\frac{1}{\left(\frac{dy}{dx}\right)}\right]\left[\frac{x^2\left(\frac{d^2y}{dx^2}\right)}{2}\right]$$

$$\therefore\ x^2\left(\frac{dy}{dx}\right) = x^3\left(\frac{d^2y}{dx^2}\right) \Rightarrow x\left(\frac{d^2y}{dx^2}\right)-\left(\frac{dy}{dx}\right)=0$$

147.(D)

$f(x)= x\log x$

$\therefore\ f'(x) = \dfrac{x}{x}+\log x = 1+\log x$

When $1+\log x = 0 \Rightarrow x = \dfrac{1}{e}$

$f''(x) = \dfrac{1}{x} \Rightarrow [f''(x)]_{x=\frac{1}{e}} = e > 0$

Thus f(x) is minimum at $x = \dfrac{1}{e}$ and

$f(x) = \left(\dfrac{1}{e}\right)\log\left(\dfrac{1}{e}\right) = \dfrac{-1}{e}$

148.(D)

$$\frac{dy}{dx} = \tan\left(\frac{y}{x}\right) + \frac{y}{x}$$

Put $\frac{y}{x} = v \Rightarrow y = xv \Rightarrow \frac{dy}{dx} = x\frac{dv}{dx} + v$

$\therefore\ v + x\frac{dv}{dx} = \tan v + v \quad \Rightarrow x\frac{dv}{dx} = \tan v$

$\therefore\ \int \frac{dv}{\tan v} = \int \frac{dx}{x}$

$\therefore\ \log|\sin v| = \log|x| + \log|c| \Rightarrow \sin v = xc$

$\therefore\ \sin\left(\frac{y}{x}\right) = xc$

149.(B)

$\tan A + 2\tan 2A + 4\tan 4A + 8\cot 8A$

$$= \tan A + 2\tan 2A + 4\tan 4A + \frac{8}{\left(\frac{2\tan 4A}{1-\tan^2 4A}\right)} = \tan A + 2\tan 2A + 4\tan 4A + \frac{8\left(1-\tan^2 4A\right)}{2\tan 4A}$$

$$= \tan A + 2\tan 2A + \frac{(4\tan 4A)(2\tan 4A) + 8\left(1-\tan^2 4A\right)}{2\tan 4A}$$

$$= \tan A + 2\tan 2A + \frac{8}{2\tan 4A}$$

$$= \tan A + 2\tan 2A + \frac{8}{2\left(\frac{2\tan 2A}{1-\tan^2 2A}\right)} = \tan A + 2\tan 2A + \frac{8\left(1-\tan^2 2A\right)}{4\tan 2A}$$

$$= \tan A + \frac{(2\tan 2A)(4\tan 2A) + 8\left(1-\tan^2 2A\right)}{4\tan 2A} = \tan A + \frac{8}{4\tan 2A}$$

$$= \tan A + \frac{8}{4\left(\frac{2\tan A}{1-\tan^2 A}\right)} = \tan A + \frac{8\left(1-\tan^2 A\right)}{8\tan A}$$

$$= \frac{\tan A(8\tan A) + 8\left(1-\tan^2 A\right)}{8\tan A} = \frac{8}{8\tan A} = \cot A$$

150.(A)

Here given plane passes through the point (2, 0, 1) and let $\bar{b} = \hat{i}$ and $\bar{c} = \hat{i} + 2\hat{j} - 3\hat{k}$.

Normal vector $\bar{n} = \bar{b} \times \bar{c} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 1 & 0 & 0 \\ 1 & 2 & -3 \end{vmatrix} = 3\hat{j} + 2\hat{k}$

The equation of plane in scalar product from is

$\bar{r}\cdot\bar{n} = \bar{a}\cdot\bar{n}$

Here $\bar{a}\cdot\bar{n} = (2\hat{i} + \hat{k})\cdot(3\hat{j} + 2\hat{k}) = 2$

$\therefore\ \alpha = 2$

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